15.1 Videos Guide

15.1a
- Volume using double integrals
  - $V = \iint_R f(x,y)\, dA,$ where $f(x,y) \geq 0$ over R, a rectangular region in $\mathbb{R}^2$

Exercise:
Calculate the iterated integral.
- $\int_0^1 \int_0^1 (x+y)^2\, dx\, dy$

15.1b
Theorem (statement):
- Fubini's Theorem: If f is continuous or has a finite number of discontinuities on $R = \{a \leq x \leq b, c \leq y \leq d\}$, then

$$\iint_R f(x,y)\, dA = \int_c^d \int_a^b f(x,y)\, dx\, dy = \int_a^b \int_c^d f(x,y)\, dy\, dx$$

- If $f(x,y) = g(x)h(y),$ then $\int_c^d \int_a^b f(x,y)\, dA = \int_a^b g(x)\, dx \int_c^d h(y)\, dy$

Exercises:
15.1c
- Estimate the volume of the solid that lies below the surface $z = 1 + x^2 + 3y$ and above the rectangle $R = [1,2] \times [0,3]$. Use a Riemann sum with $m = n = 2$ and choose the sample points to be lower left corners.
  (b) Use the midpoint rule to estimate the volume in part (a)
- Evaluate the double integral by first identifying it as the volume of a solid.
  $\iint_R (2x+1)\, dA, \qquad R = \{(x,y)|0 \leq x \leq 2, 0 \leq y \leq 4\}$

15.1d
- Calculate the iterated integral.
  $$\int_1^3 \int_1^5 \frac{\ln y}{xy}\, dy\, dx$$
- Calculate the double integral.
  $\iint_R (y + xy^{-2})\, dA, \quad R = \{(x,y)|0 \leq x \leq 2, 1 \leq y \leq 2\}$